Electromagnetic Transitions of Hyperons in a Relativistic Quark Model

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The Lorentz-covariant Bethe-Salpeter model developed by the Bonn group is used to calculate the electromagnetic form factors of the ground-state hyperons and the helicity amplitudes of the hyperon resonances. The computed magnetic moments of the ground-state hyperons agree well with the experimental values and the magnetic form factors exhibit a dipole Q^2-dependence. The photo-amplitude of the $\Lambda(1405)$ is badly reproduced, which hints at the special structure of this resonance.

1. Introduction

In describing meson electroproduction processes on the nucleon with isobar models, the implementation of electromagnetic (EM) and strong form factors constitutes one of the major sources of uncertainty. This is particularly the case for the electroproduction of kaons from the proton, where the dynamics is the result of a subtle interplay between contributions from multiple nucleon and hyperon (Λ^*) resonances. This work focuses on the computation of the EM form factors of the latter. We have used the constituent quark (CQ) model developed by the Bonn group [1,2] to calculate the EM form factors of ground-state hyperons and the helicity amplitudes of hyperon resonances. The Bonn CQ model is based on the Lorentz-covariant Bethe-Salpeter approach and is therefore well suited to describe baryon properties up to high Q^2, which involve large recoil effects [3].

2. EM Form Factors in the Bethe-Salpeter Approach

2.1. The Bethe-Salpeter Equation

The Bethe-Salpeter (BS) amplitude $\chi_{\overline{P}}$ is the analogue of the wave function in the Hilbert space of three quarks with Dirac, flavor and color degrees of freedom. Starting from the six-point Green's function, the following momentum-space integral equation for the BS amplitude can be derived :

$$\chi_{\overline{P}} = -i\, G_{0\overline{P}} \left(K^{(3)}_{\overline{P}} + \overline{K}^{(2)}_{\overline{P}} \right) \chi_{\overline{P}} \, . \tag{1}$$

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This expression incorporates all features of the model. It is Lorentz covariant by construction, and the integral kernel is the product of the free three-quark propagator $G_{0\bar{P}}$ and the sum of all three- and two-particle interactions $K^{(3)}_{\bar{P}} + \bar{K}^{(2)}_{\bar{P}}$. $G_{0\bar{P}}$ is approximated by the direct product of three free CQ propagators. We use a linear three-quark confinement potential for $K^{(3)}_{\bar{P}}$ and the 't Hooft instanton induced interaction for $\bar{K}^{(2)}_{\bar{P}}$ [2]. Both interactions are assumed to be instantaneous.

Once the BS amplitudes are known, one can calculate the matrix elements of any operator between two baryon states. When computing electromagnetic form factors, the operator of interest is the electromagnetic current operator. We use the operator $j^E_\mu = \bar{\Psi}\hat{q}\gamma_\mu\Psi$, which describes the photon coupling to a structureless CQ. Here, Ψ and $\bar{\Psi}$ are the CQ destruction and creation operators, and $\hat{q}$ is the CQ charge operator. In the c.o.m. frame of the incoming baryon ($\bar{P}' = \bar{M}$), the current matrix elements (CME's) are given by :

$$\langle\, \bar{P} \,|\, j^\mu \,|\, \bar{M} \,\rangle \simeq -3 \iint \frac{\mathrm{d}^4 \left[\frac{1}{2}(p_1 - p_2)\right]}{(2\pi)^4} \frac{\mathrm{d}^4 \left[\frac{1}{3}(p_1 + p_2 - 2p_3)\right]}{(2\pi)^4} \times \bar{\Gamma}^\Lambda_{\bar{P}}\, S^1_F(p_1) \otimes S^2_F(p_2) \otimes \left[S^3_F(p_3 + q)\, \hat{q}\gamma^\mu S^3_F(p_3)\right]\, \Gamma^\Lambda_{\bar{M}} \;, \quad (2)$$

where Γ and $\bar{\Gamma}$ are the amputated BS amplitude and its adjoint, and S^i_F is the i'th CQ propagator [3,4].

2.2. Form Factors and Helicity Amplitudes

The electromagnetic properties of composite particles are usually presented in terms of form factors, which are functions of the independent scalars of the system. The most commonly used expression for the spin-1/2 EM-vertex is :

$$\begin{aligned}\langle\, B', \bar{P}', \lambda' \,|\, j^E_\mu(0) \,|\, B, \bar{P}, \lambda \,\rangle \;&=\; e\; \bar{u}_{\lambda'}(\bar{P}')\Gamma_\mu u_\lambda(\bar{P}) \\ &=\; e\; \bar{u}_{\lambda'}(\bar{P}') \left[\gamma_\mu F^{B'B}_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_p} \kappa_{B'B} F^{B'B}_2(Q^2) \right] u_\lambda(\bar{P}) \;, \quad (3)\end{aligned}$$

where we have introduced the Dirac and Pauli (transition) form factors $F^{B'B}_1$ and $F^{B'B}_2$, and the anomalous (transition) magnetic moment $\kappa_{B'B}$. Often, the elastic form factors of the ground-state hyperons are expressed in terms of the Sachs' electric and magnetic form factors :

$$G^B_E(Q^2) = F^B_1(Q^2) - \frac{Q^2}{4M_B M_p}\kappa_B F^B_2(Q^2) = \frac{\langle B, \bar{P}', \frac{1}{2}|j^E_0(0)|B, \bar{P}, \frac{1}{2}\rangle}{\sqrt{4M^2_B + Q^2}} \quad (4)$$

$$G^B_M(Q^2) = F^B_1(Q^2) + \frac{M_p}{M_B}\kappa_B F^B_2(Q^2) = \frac{\langle B, \bar{P}', \frac{1}{2}|j^E_+(0)|B, \bar{P}, -\frac{1}{2}\rangle}{2\sqrt{Q^2}} \;. \quad (5)$$

The response of hyperon resonances to the absorption of virtual photons, $\gamma^* + Y^*(M^*) \to Y(M)$, is commonly expressed in terms of helicity amplitudes. These are directly proportional to the spin-flip ($A_{1/2}$ and $A_{3/2}$) and non-spin-flip ($C_{1/2}$) CME's, with proportionality constant $\sqrt{\frac{\pi\alpha}{2M^*(M^{*2} - M^2)}}$, where α is the fine-structure constant.

3. Results and Conclusions

Table 1
Static electromagnetic properties of the ground-state hyperons. Magnetic moments are expressed in units of μ_N, square radii in units of fm^2.

Y	$\mu_Y^{\rm exp}$ [5]	$\mu_Y^{\rm calc}$	$< r_M^2 >$	$< r_E^2 >$
Λ	-0.613±0.004	-0.61	0.40	0.038
Σ^+	2.458±0.010	2.47	0.69	0.79
Σ^0	—	0.73	0.69	0.150
Σ^-	-1.160±0.025	-0.99	0.81	0.49
$\lvert\Sigma^0 \to \Lambda\rvert$	1.61±0.08	1.52	1.96	-0.120
Ξ^0	-1.250±0.014	-1.33	0.47	0.140
Ξ^-	-0.6507±0.0025	-0.57	0.38	0.47

In Tables 1 and 2, we summarize the predictions for the static properties of the ground-state hyperons and resonances respectively. The magnetic moments are generally in very good agreement with the data. The electric mean-square radius of the Σ^- is in agreement with the values of $0.91 \pm 0.32 \pm 0.40$ fm^2 of Adamovich *et al.* [6] and $0.61 \pm 0.12 \pm 0.09$ fm^2 from Eschrich *et al.* [7]. The decay widths Γ of the hyperon resonances (Y^*'s) are poorly known. From Table 2 it is clear that for the $\Lambda(1405)$, the decay width is overestimated by more than one order of magnitude in our model. This is another indication for the peculiar structure of this resonance, which was recently already alluded to in other hadron models [8,9]. More data on the decays of hyperon resonances would clearly help in further identifying the structure of Y^*'s.

The elastic Sachs' electric and magnetic form factors of the ground-state hyperons, as well as the transition Dirac and Pauli form factors of the $\Sigma^0 \to \Lambda$ transition, are presented in Ref. [4]. There it is shown that the computed magnetic form factors can be nicely described by means of a dipole $G(Q^2) \sim \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$ with cutoffs Λ ranging from 0.79 to 1.14 GeV. We also predicted that some electric form factors change sign at a finite value of Q^2.

Table 2
Static electromagnetic properties of the hyperon resonances for which experimental results are available. Photo-amplitudes are expressed in units of 10^{-3} GeV$^{-1/2}$ and widths in units of MeV.

Y^*	$\lvert A_{1/2}\rvert$	$\lvert A_{3/2}\rvert$	$\Gamma_{\rm calc}$	$\Gamma_{\rm exp}$ [5]
$P_{13}(1385)$	62.8	108	1.46	0 – 13.9
$S_{01}(1405)$	51.5	—	0.912	0.019 – 0.035
$D_{03}(1520)$	5.50	41.2	0.258	0.0876 – 0.166

The results for the helicity amplitudes of the spin $J = 1/2$ Λ^* resonances are displayed in

Fig. 1. For the $P_{01}(1600)$, the $A_{1/2}$ reaches a maximum at a finite value of Q^2. Accordingly, our results indicate that resonances which are of minor importance in photoproduction reactions can play a major role in the corresponding electroproduction process. This also holds true for Λ resonances with higher spins [10].

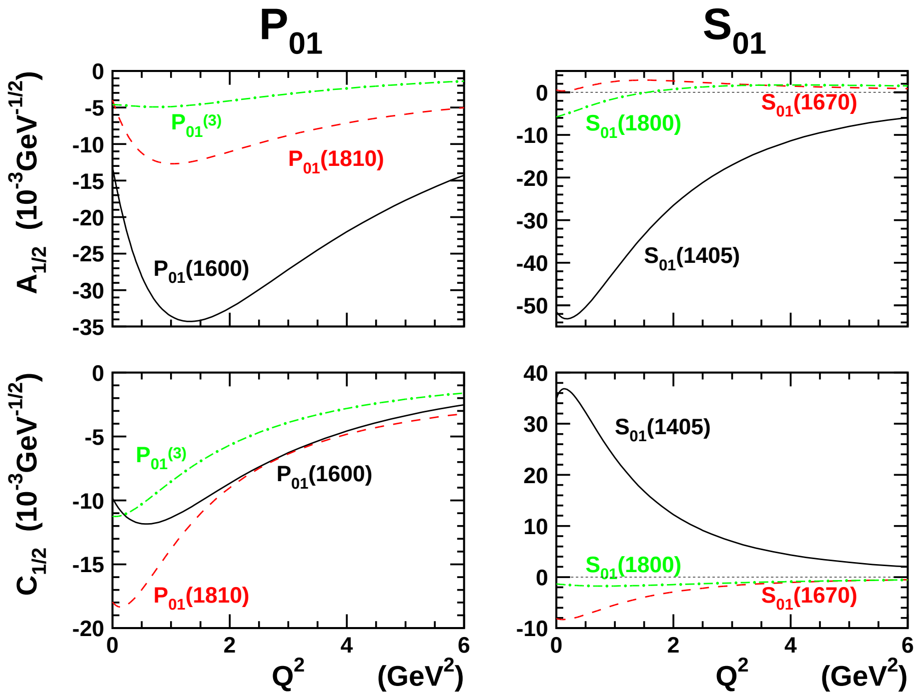

Figure 1. The Q^2 dependence of the helicity amplitudes for the three lowest-lying ($J = 1/2$, $S = -1$, $T = 0$) Λ^*-baryons with positive parity (left panels) and negative parity (right panels).

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